Mathematical Modeling Of The Output Power In Power Amplifiers

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***Abstract*—Mathematical models that describe the relationship between input and output of power amplifiers (PAs) are widely studied in the literature. Usually, the complex voltage value at the output is modeled as a non-linear function with memory of the complex voltage value at the input. The purpose of this paper is to model the output power as a non-linear function with memory of the complex voltage value at the input. One of the main points to be investigated in this work is the existence or not of a subset of terms of a given parity (even or odd) suitable for this modeling. In the theoretical study, two main forms of modeling were studied, first real model (band-pass system) and its generic version for complex numbers (low-pass system). Based on the study, it was concluded that low-pass shaping is considerably faster than band-pass and get the same result. Then we found our model equation, based on the memory polynomial (MP) and the formula of the power. After that, we performed simulations in the MATLAB. Simulating our model and comparing the predicted and wanted power, we achieved a normalized mean square error (NMSE) about -30 dB.**

***Index Terms*—Modeling, Power amplifier, Memory, Output power, Parity.**

I. INTRODUCTION

A radio frequency (RF) power amplifier (PA) is already widely used in communication systems, primarily in wireless communication, when you need to amplify a received signal before sending it. The RF PA needs to be efficient and present a linear behavior, and to do that is essential a model that at the same time does not require a lot of computational work and does not compromise its accuracy [1].

In this article, we will be using a black box system, represented in the figure 1, to model the behavior of a PA. This approach focuses on the input and output of the system and does not concerns with physical details about what is inside of the box.

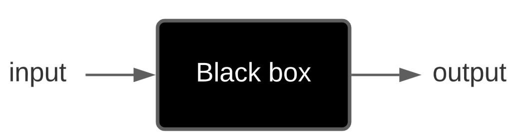

Fig. 1. Diagram of Black box system.

We have studied 2 models, band-pass and low-pass. The first is simpler and only uses real numbers. And the second is more complex and uses complex numbers, but it runs considerable faster than the first one because it adopts a lower sampling frequency, resulting in a better method to implement in our model [2].

Usually it is the output that is modeled, but recently the output power started to be modeled, a value much more important and relevant to the project and study of a PA in RF. This modeling was studied in [3], and in this article we propose a new model to predict the output power.

This article is structured with a brief review about behavioral modeling of PAs in Section II, explaining and comparing band-pass and low-pass systems. In Section III we apply the studied behavioral model to find our model, that predict its output power. In Section IV we presented the results based on the simulation of our model for different values of the memory order.

II. BEHAVIORAL MODELING OF PAS

A behavioral modeling proposes a way to get an output of a system from a database of input and output. First the model will be trained with the data that we already have, and after that, the model will be able to generalize and predict an output of the data, even when is a new input data. Our PA is represented in the figure 2.

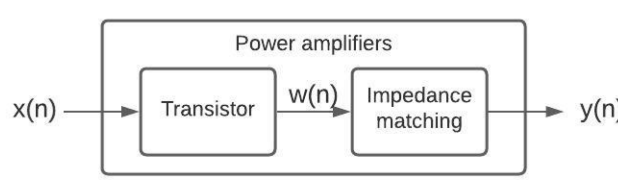

Fig. 2. Block diagram of a PA.

First we need to model the output of the system, and to do that we will use the real memory polynomial (MP) model, defined as:

$$y(n) = \sum_{p=1}^{P_0} \sum_{m=0}^{M} a_{p,m} x^p(n-m) \tag{1}$$

where $y(n)$ is the predicted output in the sample n, $x(n)$ is the input in the sample n, P_0 is the polynomial order truncation, M is the memory order and $a_{p,m}$ are the constants that will be calculated in the training.

Working with the memory polynomial equation, we can find the output by two different methods: band-pass system and low-pass system.

Beginning with a band-pass system, where we are working with only real numbers. Let's suppose the real input as a sine wave:

$$x(n) = A\cos(\omega n + \theta) \tag{2}$$

where $x(n)$ is the input of the sample n, A is a constant that represents the amplitude, ω represents the frequency and θ the phase.

We can use the MP model where $M = 0$ and $P_0 = 3$ to predict the intermediate output:

$$w(n) = a_{1,0}x(n) + a_{2,0}x^2(n) + a_{3,0}x^3(n) \tag{3}$$

where $w(n)$ is the intermediate output of the sample n, $a_{1,0}$, $a_{2,0}$ and $a_{3,0}$ are constants from our trained model.

Here we just want to verify that to every band-pass system, there is an equivalent low-pass system, therefore it would have been valid as well other values to the order of memory and polynomial. For simplicity of presentation, we have arbitrarily chosen those values.

Working with (2), (3) and after the impedance matching, we are able to find the real output in function of the input variables:

$$y(n) = \left(\frac{4a_{1,0}A + 3a_{3,0}A^3}{4}\right)\cos(\omega n + \theta) \tag{4}$$

where $y(n)$ is the output in the sample n.

The second method is the low-pass system, where we will be working with not only real numbers, but also complex numbers. In this method, we will have the following new complex input and complex output, equivalent to real input (2) and real output (4):

$$\widetilde{x} = Ae^{j\theta} \tag{5}$$

$$\widetilde{y} = \left(\frac{4a_{1,0}A + 3a_{3,0}A^3}{4}\right)e^{j\theta} \tag{6}$$

where $\widetilde{x}$ is the complex input, $\widetilde{y}$ is the complex output, e is the Euler's number and j is the imaginary unit. We can relate the real and the complex signal:

$$x(n) = \Re(\widetilde{x}e^{j\omega n}) \tag{7}$$

$$y(t) = \Re(\widetilde{y}e^{j\omega n}) \tag{8}$$

Finally, working with (5), (6), (7) and (8), we can arrive in the same conclusion that by the band-pass system in (4). This was an specific example when $P_0 = 3$ and $M = 0$, but this relation between the band-pass system and the low-pass system is valid independent of the parameters P_0 and M chosen, given some minor adaptations.

The real MP model, given in (1), is adapted to the complex MP model [4], defined as:

$$\widetilde{y}(n) = \sum_{p=1}^{P}\sum_{m=0}^{M}\widetilde{b}_{2p-1,m}|\widetilde{x}(n-m)|^{2p-2}\widetilde{x}(n-m) \tag{9}$$

where $\widetilde{y}(n)$ is the complex predicted output in the sample n, $\widetilde{x}(n)$ is the input in the sample n, P is the polynomial order, M is the memory order and $\widetilde{b}_{2p-1,m}$ are the constants that will be calculated in the training.

Finally, the relation between the polynomial order and it truncated form is given as:

$$P_0 = 2P - 1 \tag{10}$$

where P_0 is the polynomial order truncation from (1) and P is the polynomial order from (9).

In this article, we will be using the low-pass method, because it will be more computationally efficient. This happened because the real input depends of ω, that will be in the GHz order of magnitude of around, while the complex input will be around MHz. And to respect the Nyquist–Shannon sampling theorem, the lowest frequency will need lower sample rate, running faster the simulations.

III. Output Power Modeling

In general a PA model tries to predict the output based on the input, and if you need the output power, first you will need to get the output signal and then calculate its power. The goal of this paper is to find a PA model capable of predicting the output power based only in the input data. To do that we will need to apply the MP model to find the output in function of the input, and then apply to the power formula:

$$P(n) = \widetilde{y}(n)\widetilde{y}^c(n) \tag{11}$$

where $P(n)$ is the real output power in the sample n and $\widetilde{y}^c(n)$ represents the complex conjugate of the output in the sample n. Note that we are considering the resistance equal to 1 Ω.

For P up to 3 and for any M, we found our model equation:

$$
\begin{aligned}
P_{est}(n) = &\sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \widetilde{a}_{m_1,m_2}\widetilde{x}(n-m_1)\widetilde{x}^c(n-m_2) \\
&+ \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \widetilde{b}_{m_1,m_2}\widetilde{x}^2(n-m_1)\widetilde{x}^c(n-m_1) \\
&\times \widetilde{x}^c(n-m_2) \\
&+ \sum_{m_1=0}^{M}\underbrace{\sum_{m_2=0}^{M}}_{\text{if } (m1 \neq m2)} \widetilde{c}_{m_1,m_2}\widetilde{x}(n-m_1)\widetilde{x}(n-m_2) \\
&\times (\widetilde{x}^c(n-m_2))^2 \\
&+ \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \widetilde{d}_{m_1,m_2}\widetilde{x}(n-m_1)\widetilde{x}^2(n-m_2)(\widetilde{x}^c(n-m_2))^3 \\
&+ \sum_{m_1=0}^{M}\underbrace{\sum_{m_2=0}^{M}}_{\text{if } (m1 \neq m2)} \widetilde{e}_{m_1,m_2}\widetilde{x}^2(n-m_1)\widetilde{x}(n-m_2)\widetilde{x}^c(n-m_1) \\
&\times (\widetilde{x}^c(n-m_2))^2 \\
&+ \sum_{m_1=0}^{M}\underbrace{\sum_{m_2=0}^{M}}_{\text{if } (m1 \neq m2)} \widetilde{f}_{m_1,m_2}\widetilde{x}^3(n-m_1)(\widetilde{x}^c(n-m_1))^2\widetilde{x}^c(t-m_2) \\
&+ \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \widetilde{g}_{m_1,m_2}\widetilde{x}^2(n-m_1)\widetilde{x}^2(t-m_2)\widetilde{x}^c(n-m_1) \\
&\times (\widetilde{x}^c(n-m_2))^3 \\
&+ \sum_{m_1=0}^{M}\underbrace{\sum_{m_2=0}^{M}}_{\text{if } (m1 \neq m2)} \widetilde{h}_{m_1,m_2}\widetilde{x}^3(n-m_1)\widetilde{x}(n-m_2)(\widetilde{x}^c(n-m_1))^2 \\
&\times (\widetilde{x}^c(n-m_2))^2 \\
&+ \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \widetilde{i}_{m_1,m_2}\widetilde{x}^3(n-m_1)\widetilde{x}^2(n-m_2)(\widetilde{x}^c(n-m_1))^2 \\
&\times (\widetilde{x}^c(n-m_2))^3 \qquad (12)
\end{aligned}
$$

where $P_{est}(n)$ is the real output power estimated in the sample n and $\widetilde{y}^c(n)$ represents the complex conjugate of the output in the sample n, $a_{m1,m2}$, $b_{m1,m2}$, $c_{m1,m2}$, $d_{m1,m2}$, $e_{m1,m2}$, $f_{m1,m2}$, $g_{m1,m2}$, $h_{m1,m2}$ and $i_{m1,m2}$ are complex constants.

The reason why we decided to stop when P = 3 was the complexity of the model. When P = 3 and M = 1, we have an output of 6 elements, and an estimated power of 36 elements. When P = 4 and M = 1, we would have an output of 8 elements, and an estimated power of 64 elements. It would take considerably longer to calculate, analyze, organize and generalize.

In (12), we can observe that we have elements of only an even parity, 2 inputs in the first summation, 4 inputs in the second and third summations, 6 inputs in the fourth, fifth and sixth summations, 8 inputs in the seventh and eighth summations and 10 inputs in the tenth summation. Besides that, each element has the same number of inputs and complex conjugate inputs.

IV. Results

In this section we will simulate the model (12) using the software MATLAB, with floating point double precision. We will use the data collected from a PA in GaN tecnology, class AB. It was used a Rohde & Schwarz FSQ vector signal analyzer (VSA) having the sampling frequency of 61.44 MHz. The PA was excited by a carrier signal of frequency 900 MHz and modulated by a 3GPP WCDMA envelope signal having about 3.84 MHz of bandwidth [5].

The data already has been separated in extraction (3221x1) and validation (2001x1) to prevent any extrapolation problems. And given the amount of data, to solve a system of linear equations, we used the function mldivide (\) instead of the matrix inverse (inv), avoiding ill-conditioning.

First we used the extraction data to train our model, and the validation data to calculate the estimated output power. With this result, we compared the wanted output power applying the output validation data to (11) and getting its real part. To the comparison, we are using the NMSE (Normalized Mean Square Error) [6], defined as:

$$
NMSE_{dB} = 10\log\left(\frac{\sum_{n=1}^{N}|y_{ref}(n) - y_{test}(n)|^2}{\sum_{n=1}^{N}|y_{ref}(n)|^2}\right) \qquad (13)
$$

where N is the length of the vector, y_{ref} is the vector of reference, in this case the wanted output power, and y_{test} is the vector that is being tested, the estimated output power.

We are using NMSE instead of MSE (Mean Square Error), because it will give a normalized value, adjusting the error to the magnitude of the used data. And for the same reason, the NMSE is more adopted in this field of PA, facilitating performance comparisons.

Based on what was written, we simulated our model for different values of M and get the results compiled in the table I.

TABLE I
Results of the Proposed Output Model for M from 1 to 5

(P; M)	NMSE (dB)	Time (s)
(3; 1)	-21.66	01.61
(3; 2)	-29.08	05.89
(3; 3)	-31.46	11.56
(3; 4)	-32.79	18.88
(3; 5)	-33.21	27.22

We can notice that according to M grows, the complexity of the model increases as well, and we expected to get a lower NMSE until arrive in some limit. Based on the result, we assume that this limit is between M = 2 and M = 3, because after that the NMSE gets slowly smaller and does not compensate the needed extra time.

Finally, to visualize the result, we can plot the Q-Q (Quantile-Quantile) graph of the wanted and predicted power in the same graph to make a comparison. We are going to plot for M = 3. No clear distinction between wanted and estimated values is observed.

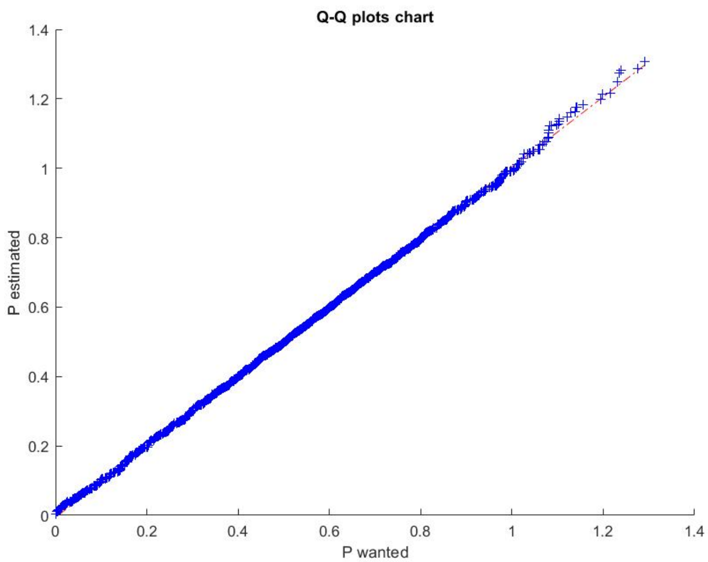

Fig. 3. Q-Q chart when M = 3.

V. Conclusion

This work introduced a model that directly estimated the output power as a memory polynomial of the complex input. Based on the reported results of the simulation, we were able to validate our model and get an NMSE between - 20 dB and - 30 dB, depending of the memory order. In general terms, we could say that if you need a model fast to simulate, you can use the M = 2, and if you need the result with a better accuracy, M = 3 should be good and not take much time.

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